March 14, 2014

MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 3 May 2012 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	3	0	3

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

SECTION A

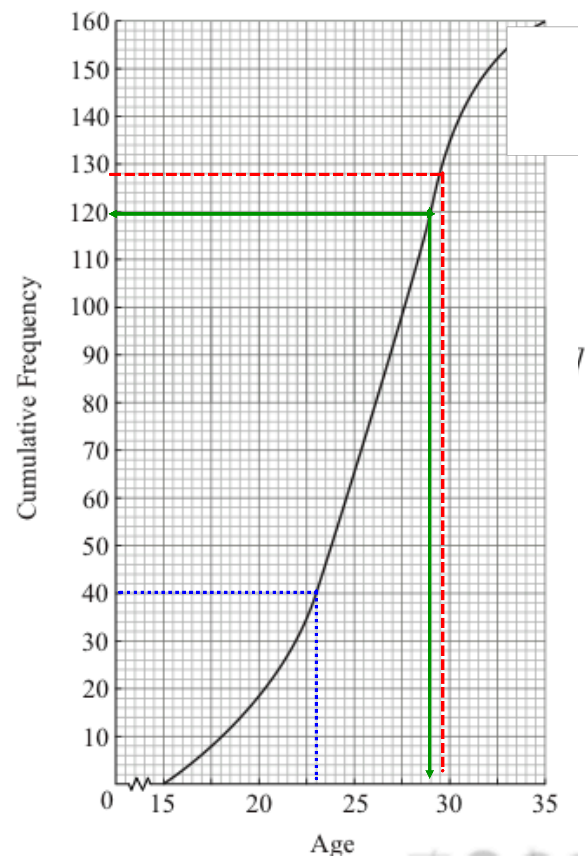
*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

The ages of people attending a music concert are given in the table below.

Age	$15 \le x < 19$	$19 \le x < 23$	$23 \le x < 27$	$27 \le x < 31$	$31 \le x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	p	160

(a) Find p. *[2 marks]*



(b) Use the diagram to estimate

(i) the 80[th] percentile;

(ii) the interquartile range.

2. *[Maximum mark: 6]*

Let $\boldsymbol{A}$ be a 2×2 matrix and $\boldsymbol{B}$ an $m\times n$ matrix, where $\boldsymbol{A}=\begin{pmatrix}-2 & 0\\ 1 & 3\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}1 & 0 & 3\\ -1 & 3 & 0\end{pmatrix}$.

(a) Write down the value of m and of n. *[2 marks]*

(b) Find $\boldsymbol{AB}$. *[3 marks]*

(c) Let $\boldsymbol{C}$ be a $p\times4$ matrix. Given that the product $\boldsymbol{BC}$ exists, write down the value of p. *[1 mark]*

3. *[Maximum mark: 6]*

Let $f(x) = e^{6x}$.

(a) Write down $f'(x)$. *[1 mark]*

The tangent to the graph of f at the point $P(0, b)$ has gradient m.

(b) (i) Show that $m = 6$.

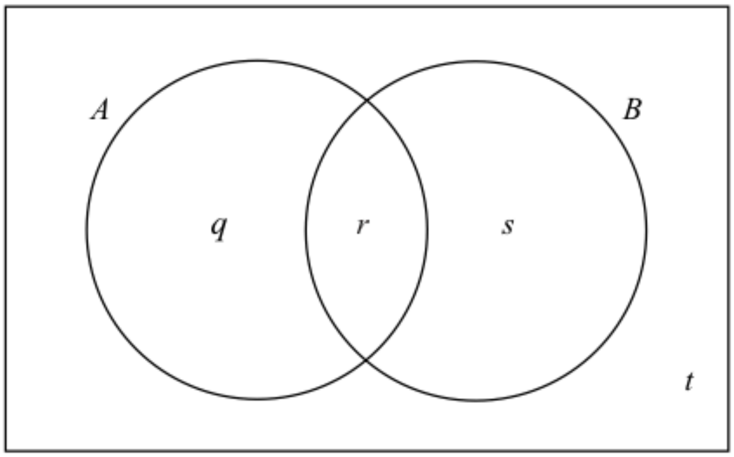
(ii) Find b. *[4 marks]*

(c) Hence, write down the equation of this tangent. *[1 mark]*

March 14, 2014

4. *[Maximum mark: 7]*

Events A and B are such that $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.7$.



The values q, r, s and t represent probabilities.

(a) Write down the value of t. *[1 mark]*

(b) (i) Show that $r = 0.2$.

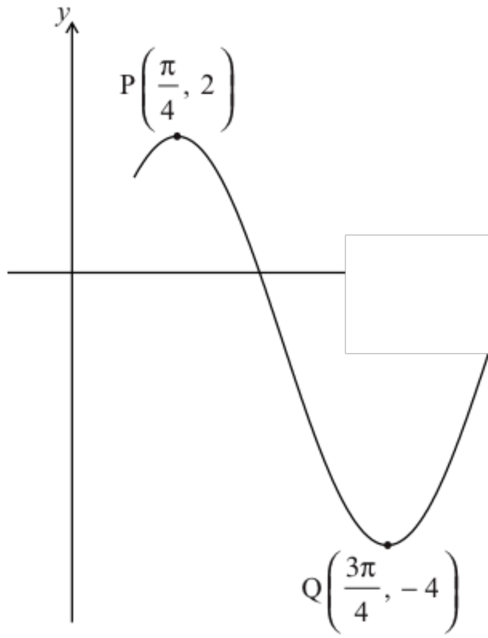
(ii) Write down the value of q and of s. *[3 marks]*

(c) (i) Write down $\mathrm{P}(B')$.

(ii) Find $\mathrm{P}(A|B')$. *[3 marks]*

5. *[Maximum mark: 7]*

The diagram below shows part of the graph of $f(x) = a\cos(b(x-c)) - 1$, where $a > 0$.



The point P$\left(\frac{\pi}{4}, 2\right)$ is a maximum point and the point Q$\left(\frac{3\pi}{4}, -4\right)$ is a minimum point.

(a) Find the value of a. *[2 marks]*

(b) (i) Show that the period of f is π.

(ii) Hence, find the value of b. *[4 marks]*

(c) Given that $0 < c < \pi$, write down the value of c. *[1 mark]*

6. *[Maximum mark: 6]*

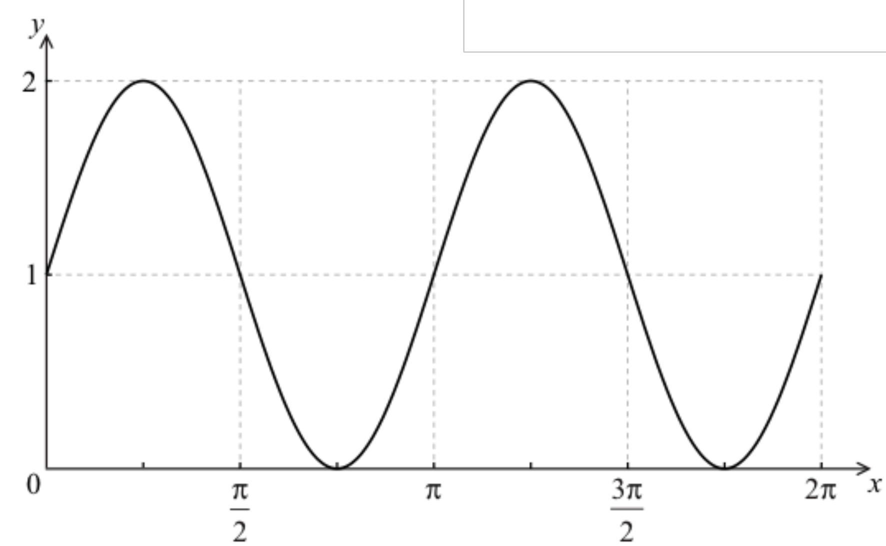
Given that $\int_0^5 \frac{2}{2x+5}\,\mathrm{d}x = \ln k$, find the value of k.

7. *[Maximum mark: 6]*

Let $f(x) = (\sin x + \cos x)^2$.

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$. *[2 marks]*

The graph of f is shown below for $0 \le x \le 2\pi$.



(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$. *[2 marks]*

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$.

(c) Write down the value of p and a possible value of k. *[2 marks]*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

A line L_1 passes though points $\mathrm{P}(-1, 6, -1)$ and $\mathrm{Q}(0, 4, 1)$.

(a) (i) Show that $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

(ii) Hence, write down an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[3 marks]*

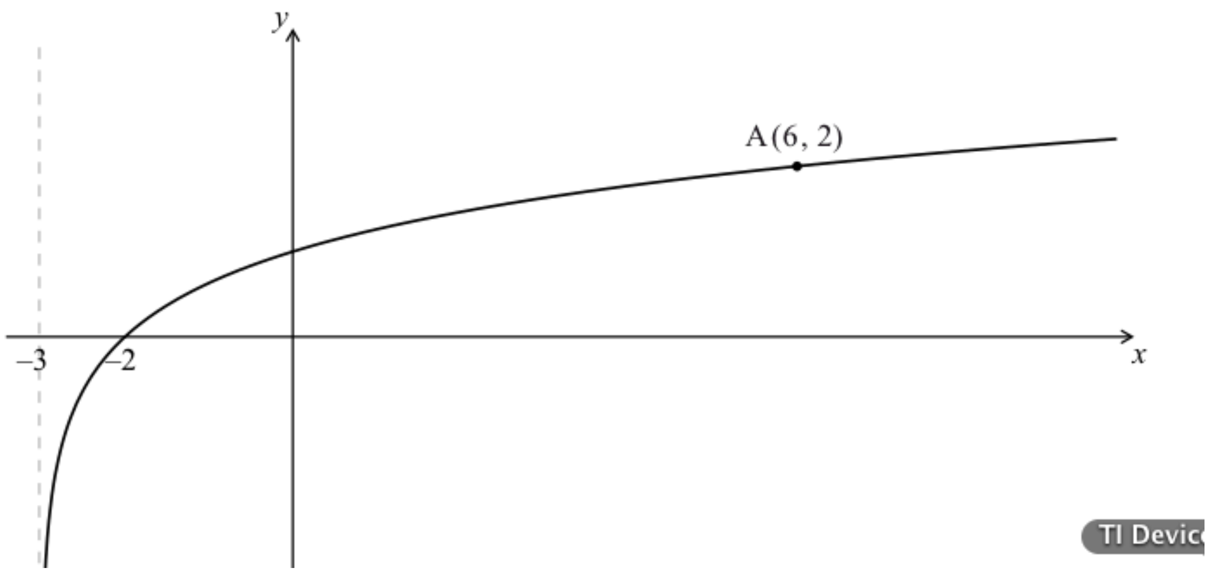
A second line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

(b) Find the cosine of the angle between $\overrightarrow{\mathrm{PQ}}$ and L_2. *[7 mar*

(c) The lines L_1 and L_2 intersect at the point R. Find the coordinates of R. *[7 mar*

9. *[Maximum mark: 13]*

Let $f(x)=\log_p(x+3)$ for $x>-3$. Part of the graph of f is shown below.



The graph passes through $A(6, 2)$, has an x-intercept at $(-2, 0)$ and has an asymptote at $x=-3$.

(a) Find p. *[4 marks]*

The graph of f is reflected in the line $y=x$ to give the graph of g.

(b) (i) Write down the y-intercept of the graph of g.

(ii) Sketch the graph of g, noting clearly any asymptotes and the image of A. *[5 marks]*

(c) Find $g(x)$. *[4 marks]*

10. *[Maximum mark: 15]*

In this question, you are given that $\cos\frac{\pi}{3} = \frac{1}{2}$, *and* $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

The displacement of an object from a fixed point, O is given by $s(t) = t - \sin 2t$ for $0 \le t \le \pi$.

(a) Find $s'(t)$. *[3 marks]*

In this interval, there are only two values of t for which the object is not moving. One value is $t = \frac{\pi}{6}$.

(b) Find the other value. *[4 marks]*

(c) Show that $s'(t) > 0$ between these two values of t. *[3 marks]*

(d) Find the distance travelled between these two values of t. *[5 marks]*